Week 0

- 1. If f(x)=2x+20/x-2, g(x)=2x-3, then f(g(3)) is c. 26
- 2. How many three digit numbers greater than 500 can be formed using numbers 1,3,5,7,9, without repetition? b. 36
- 3. The coefficient of x^4y^2 in the expression $(4x^2y+24)^3$ is equal to b. 1152
- 4. Let P be set of prime numbers and Z be set of integers, then $P \cap Z$ will be?

c.
$$P \cap Z = \{2,3,5,7,11,13,17,19....\}$$

- 5. Which of the following is an empty set?
- a. {x: x is a even prime greater than 3}
- 6. Which of the following is a rational number(s)?
- d. √64/√25

- 7. A dice is thrown in the air. The probability of getting odd numbers
- a. 1/2
- 8. The sum of two numbers is 27 and the product is 182. The numbers
- c. 13 and 14
- 9. Let f:R→R be defined by $f(x) = \{ 2x : x > 3 ; x^2 : 1 < x < 3 ; 3x : x < = 3 \}$ Then f(-1)+f(2)+f(4)a. 9
- 10. If x + 1/x = 2, then what will be the value of $x^7 + 1/x^7$? b. 2

Week 1

- 1. In how many ways the word 'PATHOGEN' can be arranged such that letter O always comes to the left of K.
- c. 20160
- 2. Rahul has 7 differently colored shirts, 5 different jeans, 2 pairs of shoes and 3 different caps. In how many ways can Rahul dress up before going to a party?



- 3. In how many ways can the word 'DOCUMENTATION' be arranged so that all the consonants come together?
- d. 6!/2!×7!/(2!×2!)
- 4. A question paper consists of 20 questions, having 2 parts. Each part consists of 10 questions. Students have to answer 12 questions in total, of which at least 4 questions should be from part 1 and at least 4 questions from part 2. In how many ways can the student select questions?
- c. 123480
- 5. What is the coefficient of the 6^{th} term in the expression ($5x^4/13 x^3/2$)⁷
- b. -525/5408
- 6. Which of the following is NOT an application of catalan numbers?
- **b.** Counting the number of ways in which a person can choose a red ball from a bag containing 10 black balls, 5 red balls and 15 blue balls.
- 7. There are 2 vacant positions for the president and a vice president of a club. The post can be given to any of the 50 members of the club and a member can take more than one cabinet position. In how many ways can the 2 vacant positions be filled?

a. 50^2

- 8. A bag contains five black balls, seven white balls, and nine red balls. The number of ways in which three balls can be drawn from the bag so that all the three drawn balls will have different colors is d. 315
- 9. What is the total number of paths from (-3,-2) to (5,6) without crossing the line passing through (-3,-2) and (5,6) (i.e., without crossing the diagonal)?

d. 4862

10. Evaluate $({}^{20}C_4 \times {}^{21}P_2) / {}^{100}C_{99}$ b. 20349

Week 2

1. If A and B are sets and $A \cup B = A \cap B$, then which of the following is(are) true?

c. A = B

2. If A = $\{3,3,4,5,6,6,9\}$, B = $\{\{5,9\},12,\{1,\{x,y\}\},\{a,b,c\}\}\}$, C = $\{\phi\}$, X = $\{x,y,z\}$ and $Y = \{A,B\}$. Which statement is true? b. Cardinality of B is 4

3. Let $A = \{3,5,6,7,8,9\}$, $B = \{2,4,5,11,14\}$ and $C = \{-1,0,1\}$. Which of the following is not a disjoint set?

d. A and B

4. Let set $A = \{4,5,6,7,9\}$ and P(A) denote the power set of A. What is the cardinality of P(P(A))?

b. 2³²

5. Let A = $\{x \mid x \in Z; -2 \le x \le 5\}$ and B = $\{x \mid x \in N; x \text{ is a prime number}\}$, what is A-B?

c. {-2,-1,0,1,4}

6. In a group of 120 people, 50 people love to eat dairy milk chocolate, 70 people love to eat Kitkat, and 20 people do not like any type of chocolate. How many people love to eat both types of chocolate? a. 20

7. Which of the following pairs of sets are equal?

c. $A = \{-2, -3\}, B = \{x: x \text{ is a Solution of } x2+5x+6=0x2+5x+6=0\}$

8. Suppose $A = \{1,2,3,4\}$. How many subsets of 2 distinct elements are possible?

b. 6

9. Simplify (A^c∩B^c)^c ∪ B

d. A^c ∪ B

10. The set O of odd positive integers less than or equal to 10 is **b.** {1,3,5,7,9}

Week 3

- 1. Let p: Ram likes ice cream or Sudha likes toffee, Negation of the statement p is?
- a. Ram does not like ice cream and Sudha does not like ice-cream
- 2. Which of the following statement(s) is/are true?
- I. The sentence, "Rajesh is a hardworking man" is a statement
- II. 9 is not a prime number
- III. Cardinality of $A \cup B$ is 5, where, $A = \{1,2,3\}$ $B = \{2,7,8\}$
- b. I, II and III
- 3. If A is any statement, then which of the following is a tautology? b. A V ¬A
- 4. "It is not that I don't like travelling", said Reena, what does Reena mean?
- a. Reena likes travelling less frequently
- 5. (p \rightarrow r) \vee (q \rightarrow r) is logically equivalent to d. $(p \land q) \rightarrow r$

6.
$$(\neg p \lor q) \lor (\neg q \land \neg p)$$
 is equivalent to

7. If the truth value of s is True, where s is $\neg((\neg q \land \neg p) \rightarrow p)$ then the truth value of p and q are respectively?

8.
$$\neg(p \lor q) \lor (\neg p \land \neg q)$$
 is logically equivalent to d. $\neg p \land \neg q$

- 9. What can we infer from the below statement?
- q: Gautam will go for a long drive to Mahabaleshwar or he will play cards
- r: Gautam did not go to Mahabaleshwar
- b. Gautam will play cards

10. If $(\neg p \rightarrow \neg q)$ is false then what is the truth value of p and q?

Week 4

- 2. A = {srijit, akash, abhi} and B = {shraddha, sanchita} Which of the following subsets belong to $A \times B$?
- a. {(srijit, sanchita), (abhi, shraddha), (akash, sanchita), (srijit, shraddha)}
- 3. What is the total number of reflexive relations of the set {5,7,13,15}? d. 4096
- 4. $S = \{1,2,3,4,5\}$. A relation R on set S is defined as $R = \{(b,a) \mid 0 \le -a\}$ + $b \le 3$ } What is the cardinality of set R? c. 14
- 5. Let R be a relation on a collection of sets defined as follows, $R = \{(A,B) \mid A \subseteq B\}$

Which of the following statement(s) is/are correct?

- a. R is reflexive and transitive
- c. R is anti-symmetric
- 6. Let a relation R be defined as $R = \{(A, B) \mid Both A \text{ and } B \text{ live in the } A \text{ and }$ same city). Pick out the correct statement(s).
- **b.** *R* is reflexive
- c. R is transitive
- d. R is symmetric

- 7. Which of the following is an equivalence relation?
- **b.** $R = \{(x,y) \mid y x = 0\}$
- 8. Suppose the cardinality of a set A is 4 and the cardinality of a set B is 3, what are the cardinalities of the cartesian product $\mathbf{A} \times \mathbf{B}$ and the power set of A × B?
- c. 12 and 4096
- 9. Which of the following collection of subsets is a partition of A ={1,2,3,4,5}?
- d. {1,2}{5}{3,4}
- 10. Let A be a set with cardinality m, and B be a set with cardinality m. There are a total of 64 symmetric relations on A, and 216 antisymmetric relations on B. What is $n \cdot m$?
- a. 9

Week 5

- 1. Which of the following is(are) true for the given function?
- $f: R \rightarrow R$
- $f(x)=x^2+2$ where, R is a set of real number
- a. f is not injective
- **d.** *f* is not surjective
- 2. Consider the following table:

Name of students	Height (in cm)
Rohit	168
Amit	165
Krishna	169
Atharva	170
Nitin	165

We can think of this as a function f from the set of students to the set of integers between 160 and 170. Now pick out the correct statement from the following.

 ${f d}.\ f$ is neither one to one nor onto

3. Let
$$f: R \rightarrow R$$
 such that $f(x)=x/2+7$ b. f is bijective

4. If a function is defined as
$$f(x)=2x+15$$
 then the value of $f^{-1}(25)$ is b. 5

5. Set ${\it C}$ has cardinality p and a total of 5040 bijective functions. What is the value of p^2 ?

d. 49

6. find the domain and range of the following real-valued function. $f(x) = \sqrt{3-x}$

```
d. domain= \{x \in \mathbb{R} \mid x \leq 3\}
range= \{x \in \mathbb{R} \mid x \ge 0\}
```

7. If f and g are function from R to R and $f(x)=3x^2+x-13$ and g(x)=20/(3x+8) then $f \circ g$ (12) is. c. -1443/121

8. Let us define a function $f: Z \rightarrow Z$ as follows, $f(x) = \{x/2 \text{ if } x \text{ is even, } 0 \text{ if } x \text{ is odd} \}$ Z is a set of integers.

a. onto but not one-to-one

9. The relation R is defined as $R = \{(x, y): x, y \in \mathbb{N}, x + y = 5\}$ then the range is?

d. {1,2,3,4}

10. Let A be set with cardinality m, there are a total of 3024 one to one function from A to B, what are the values of n and m respectively?

b. 4 and 9

Week 6

1. There are n people in a town, having the length of their name greater than 3. What is the number of people in the town such that you are guaranteed to find at least two people with the same second alphabet in their name?

d. 27

2. Given a set of integers from 51 to 250, { 51, 52, 53,.....,250}. What is the least number of integers that can be selected such that any two of the chosen integers are consecutive?

We can think of this as a function f from the set of students to the set of integers between 160 and 170. Now pick out the correct statement from the following.

b. 101

3. Which of the following statements is FALSE?

b.
$$n^2 > 2n + 1 \forall n \in \mathbb{Z}^+$$

4. Is the given inequality true?

$$2^n < n! \forall n \in N$$

b. No

- 5. A black-coloured (opaque) bag contains 15 red gloves and 15 green gloves, Krishna takes out one glove at random each time. How many gloves must he take out to be sure that he has at least 2 red gloves? a. 17
- 6. How many of the following statements are true?

c. 2

7. For positive integer n, $10^{n-2} > 81n$, if d. n≥5

8. A basket containing 60 walnuts was kept in the veranda, and 10 squirrels randomly came and took away some walnuts. After some time, the gardener found the basket empty. Which of the following conclusions is always true for the above-given situation?

b. At most one squirrel went back with at least 6 walnuts

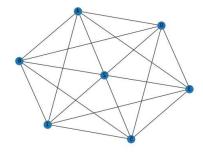
- 9. State whether the following statement is True/False: For all $n \in \mathbb{N}$, $6^n - 1$ is a multiple of 5.
- a. True

10. Madhu, Anushka, and Prasad bought a White Chocolate consisting of n blocks, the chocolate is distributed equally among all the three members. Madhu likes to enjoy the chocolate, having it one block at a time. How many total breaks does Madhu require to finish her share of chocolate?

d. n/3 - 1

Week 7

1. Which of the following graphs are not complete graphs?



b

e

2. What is the degree sequence of the given graph?

c. (4,4,3,3,3,5,2)

3. What are the cut edge and the cut vertex respectively, in the following graph?

c. (B, E) and E

4. The number of components in a K_n and C_n respectively are?

d. 1,1

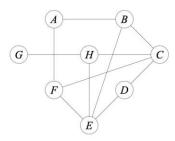
5. Which of the following is a graphic sequence?

a. 5,3,3,2,2,1

6. Which of the following is not a path from A to H?

K

KUIZZER DISCRETE MATHEMATICS WEEK 0-12



a.
$$\{A - B - E - F - G - H\}$$

d.
$$\{A - F - C - B - E - D - C - H\}$$

$$f. \{A - H\}$$

7. If an edge is removed from a cycle in a graph, then the graph becomes disconnected.

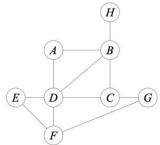
State whether true/false.

b. False

8. For a simple graph with vertices, how many subgraphs can be constructed, such that the subgraph is an induced subgraph as well as a spanning subgraph?

a. 1

9. Observe the following graph.



Choose the correct option(s) from below.

a.
$$\{D - E - F - G - C - D\}$$
 is a cycle

b.
$$\{H - B - D - C - G - F - D - A\}$$
 is a trail

e.
$$\{A - B - D - C - G - F - D - A\}$$
 is a circuit.

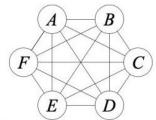
- 10. Which of the following statements is/are true?
- I) If there is a walk from P to Q then, there must be a path from P to Q.
- II) The number of edges in a tree is equal to one less than the number of vertices.
- III) Every graph has an odd number of odd-degree vertices.
- d. I and II

Week 8

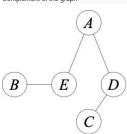
1. What is the number of colors required to color a complete graph containing n vertices?

d. n

2. Which of the following graphs does not have a Eulerian circuit?



Complement of the graph



3. What is the number of edges for a connected planar graph	having 8	3
vertices, and 6 regions?		

a. 12

- 4. Which of the following graphs can be concluded to be Hamiltonian?
- a. A graph with the number of edges of every node, greater than n/2 d. k_6
- 5. Find the total number of edges in the complement graph ${\it Gc}$, where ${\it G}$ has 12 vertices and 34 edges.

b. 32

- 6. Which of the following statement(s) is/are true?
- I) We can find a tree that is not planar.
- II) We can conclude that a graph is connected if the degree of all the vertices is

greater than equal to n/2.

III) Given a graph G, there is a cycle of length k and k is less than equal to nn, then we can find a path of length at least k+1.

d. II and III

- 7. A graph where we can traverse through all the vertices, without repeating edges or vertices more than once is called?
- c. Hamiltonian graph



8. State whether true/false:

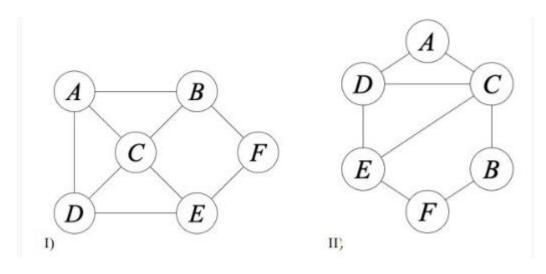
A bipartite graph can have an odd cycle.

b. False

9. What is the cardinality of the set of edges of the complement of a complete graph ${\it G}$ having 5 vertices?

c. 0

10. What is the chromatic number of the graph given below respectively?



d. 3,3

Week 9

1. What is the closed-form expression of the generating function for the sequence: 7,0,7,0,7,0.....?

c. $7/(1-x^2)$

- 2. What is the coefficient of x^4 in $(1-x)^{-5}$?
- a. 70
- 3. What is the sequence in the generating series for the expression $5x^2+10x^3+15x^4+20x^5+\cdots$?
- d. 0, 0, 5, 10, 15, 20,...
- 4. What is the generating function of the sequence 0, 1, 2, 3, 4,?
- a. Derivative of the function $(1+x+x^2+x^3+x^4+\cdots)$
- 5. In how many ways can 200 players be distributed among 10 teams, so that each team gets at least 7 players and not more than 12?
- d. The coefficient of x^{200} in $(x^7+x^8+x^9+x^{10}+x^{11}+x^{12})^{10}$
- 6. How many 5 lettered words can be formed from the following words, given that you have to pick one letter per line?

- 7. A bag contains several blue, green, purple and yellow candies, in how many ways can Amit eat 8 candies from these blue, green, purple and yellow-colored candies?

b. 165

8. Find the closed form expression of the generating function for the sequence: 1, 11, 121, 1331, 14641.....?

b. 1/(1-11x)

9. State whether true/false:

 $(1+x)^6$ is the generating function for the sequence:

a. True

10. The sequence generated by the function $(x+1)/(1-x)^3$.

c. 1^2 , 2^2 , 3^2 , 4^2 , 5^2 ,...

Week 10

1. How many positive integers between 1 and 1000, (including both 1 and 1000) are not divisible by 3, 5, and 7?

d. 457

2. How many onto functions are possible from a set of 9 elements to a set of 4 elements?

c. 186480

3. In how many ways can integers from 1 to 10 be arranged such that no prime number is at its natural place?

$$10! - \binom{4}{1} 9! + \binom{4}{2} 8! - \binom{4}{3} 7! + \binom{4}{4} 6!$$

C

- 4. In a 3×3 chessboard:
- I) One rook can be placed in 9 ways.
- II) Two rooks can be placed in (9/2) ways such that they do not kill each other (non-taking rooks).
- III) Three rooks can be placed in 6 ways such that they do not kill each other (non-taking rooks).

Which of the following statements are true?

b. I and III

5. How many integer solutions are there for the equation x+y+z=18, where, x<5,y<9,z<7?

a. 1

7. A basketball team has 7 players, each having distinct names. Every player has a jersey with their names on the back of it. If the jerseys are lying in the changing room, turned inside-out. In how many ways can the players wear them so that no one wears the jersey of their name (each one wears someone else's jersey)?

c. 1854

8. In a class of 120 students, 67 students like Mathematics and 86 students like Physics, while 53 students like both the subjects. What is the total number of students who like neither of the two subjects? b. 20

9. In how many ways can 5 tigers, 5 lions, and 5 elephants march-past, in the same row, such that no consecutive 5 animals of the same family are together?

$$\frac{15!}{(5!)^3} - 3(\frac{11!}{(5!)^2}) + 3(\frac{7!}{(5!)}) - 3!$$

10. For some Greek alphabets α , β , γ , δ ,... (n-1)th alphabet, (n)th alphabet, there are 491310 derangements, where α , β , γ , δ , ε , ζ appear in the first 6 positions. What is the value of n?

d. 13

Week 11

1. Which of the following are true?

 $500n^2 < 1500n$

 $5000n^2 > n^3$

 $300n > 3n^2$

 $999n^2 < 9n^3$

 $d. 999n^2 < 9n^3$

- 2. How many n-bit binary strings are possible without having consecutive ones?
- **c.** T(n)=T(n-1)+T(n-2) such that T(1)=2 and T(2)=3
- 3. Let a_n be a sequence that satisfies the recurrence relation $a_n = 20a_{n-1}$
- $_{1}$ 25 $_{a_{1}-2}$ where a_{0} = 4 and a_{1} = 14. What are a_{3} and a_{4} respectively?
- **b.** 3250, 60500
- 4. Which of the following statement(s) is/are true?
- I) The solution for the tower of Hanoi is 2ⁿ 1.
- II) The recurrence relation for the tower of Hanoi is T(n)=2T(n-1)+1 for $T_1=1$.
- III) Complexity of tower of Hanoi is n².
- c. I and II
- 5. Which of the following sequence is correct?

$$O(log(n)) < O(log(n^2)) < O(n) < O(nlog(n)) < O(n^2)$$
 $O(log(n)) < O(log(n^2)) < O(n) < O(n^2) < O(nlog(n))$
 $O(n) < O(n^2) < O(log(n)) < O(log(n^2)) < O(nlog(n))$
 $O(n) < O(n^2) < O(log(n)) < O(nlog(n)) < O(log(n^2))$
 $O(log(n^2)) < O(nlog(n)) < O(log(n)) < O(n) < O(n^2)$
a. $O(log(n)) < O(log(n^2)) < O(nlog(n)) < O(nlog(n)) < O(n^2)$

- 6. Given the sequence {0, 1, 1, 2, 3, 5, 8, 13......} find is the 23rd term this sequence.
- d. 28657

7. Paras is reading a novel that consists of 1000 pages. While he reads the book, his dad calls him for some work. Being in a hurry, he forgets to put the bookmark on the page he was reading, though he remembers the page number. How much minimum effort would Paras require to get back to the page that he last visited, if he uses an efficient searching technique?

c. log(1000)

8. Which of the following is the correct recurrence relation for climbing 20 stairs? Climbing stairs simply means walking on stairs, covering one stair in one step.

$$d. a_{20} = a_{18} + a_{19}$$

9. What can we say about the following function?

$$egin{aligned} f_1(n) &= 40n^3 \ f_2(n) &= 50n^2 \ f_3(n) &= 23n^2 + 20n \ f_4(n) &= 18n^3 + 5n^2 + 9n \end{aligned}$$

- c. f₄(n) is an order n³ function
- d. $f_2(n)$ and $f_3(n)$ grow slower than $f_1(n)$

10. What is the solution of the recurrence relation $a_n = 3a_{(n-1)} - 10a_{(n-2)}$? given that $a_0=1$ and $a_1=2$.

$$a_n = \left(\frac{20}{7}\right)^n - \left(\frac{6}{7}\right)^n$$

Week 12

- 1. Which of the following are true?
- (Z,×) is a group
- (Z,+) is not a group
- (Q,×) is a group
- (Z,-) is not a group
- (R,+) is a group
- d. (Z,-) is not a group
- e. (R,+) is a group
- 2. Let T be a group of 100 elements, what is the largest possible subgroup of T other than T itself?
- a. 50
- 3. The generating function for odd partitions of x is:

$$rac{1}{(1-x)} imesrac{1}{(1-x^3)} imesrac{1}{(1-x^5)} imes\ldots$$

d

4. For a path graph of P₇, in how many ways can you color this graph with 4 colors so that no two adjacent nodes have the same color? b. 2916

- 5. What are the elements of Z_7 , i.e., integer modulo 7?
- c. {0, 1, 2, 3, 4, 5, 6}
- 6. Which of the following statement(s) is/are true?
 - I) The operations of a group and its subgroup can be different.
- II) (Q, +) is a subgroup of (R, +)
- III) A group under addition modulo n, where n is prime, cannot have a subgroup except for a trivial subgroup identity element alone.
- a. II and III
- 7. State whether true/false:

 $P_d(n)$ is the coefficient of x^n in the function $(1+x)(1+x^2)(1+x^3)....(1+x^n)$

- a. True
- 8. Let A be a group with 70 elements. Then A cannot have a subgroup of cardinality
- c. 20
- 9. $(Z_6,+)$ is a group, what is the inverse of 4 in this group?
- b. 2
- 10. Given a complete graph with 11 vertices, in how many ways can you color this graph such that no two adjacent vertices have the same color? You are given 26 colors.

c.
$$C(K_{11}) = 26 \times 25 \times 24 \times ... \times 16$$